Key

Birzeit University
Mathematics Department
Math332
Quiz 2

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Name:.. | **Number:**..............

Section: | **Date: 25/09/2019**

Important: Do only TWO questions.

Question I [5 points]. Find all values of the parameters α and β, if any, for which the functions $f(x) = \alpha x + \beta$ and $g(x) = e^x$ are orthogonal over $[0, 1]$.

Question II [5 points]. Assume that f is a continuous piecewise function of period 2π on $[-\pi, \pi]$. Let

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\, nx + b_n \sin\, nx\right],$$

then prove the following **Parseval's identity**

$$\frac{1}{2}a_0^2 + \sum_{n=1}^{\infty} (a_n^2 + b_n^2) = \frac{1}{\pi}\int_{-\pi}^{\pi} f^2(x)dx.$$

Question III [5 points]. If the Fourier series of

$$f(x) = \begin{cases} 0 & , \quad -\pi < x < 0 \\ \sin x & , \quad 0 \leq x < \pi \end{cases}$$

is given by

$$f(x) = \frac{1}{\pi} + \frac{1}{2}\sin x + \frac{1}{\pi}\sum_{n=2}^{\infty}\left[\frac{1 + (-1)^n}{1 - n^2}\cos nx\right].$$

Show that

$$\frac{1}{2} + \frac{1}{1\cdot 3} - \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} - \frac{1}{7\cdot 9} + ... = \frac{\pi}{4}.$$

Good Luck

<u>Question I.</u> $\int_0^1 f(x)\, g(x)\, dx = 0 \Rightarrow \int_0^1 (\alpha x+\beta)\, e^x dx = 0$

$$\Rightarrow \left[(\alpha x+\beta)e^x - \alpha e^x\right]\Big|_0^1 = 0$$

$$\begin{array}{cc} \alpha x+\beta & e^x \\ \alpha & e^x \\ 0 & e^x \end{array}$$

$(+)$ $(-)$

$$\Rightarrow \left[(\alpha+\beta)e - \alpha e\right] - \left[\beta - \alpha\right] = 0$$

$$\Rightarrow (e-1)\beta + \alpha = 0 \quad \text{or} \quad \boxed{\alpha = (1-e)\beta}$$

<u>Question II.</u> multiply both sides of (*) by $f(x)$,

$$f^2(x) = \frac{a_0}{2} f(x) + \sum_{n=1}^{\infty}\left[a_n f(x) \cos(nx) + b_n f(x) \sin(nx)\right]$$

$$\Rightarrow \int_{-\pi}^{\pi} f^2(x)\, dx = \frac{a_0}{2}\int_{-\pi}^{\pi} f(x)\, dx + \sum_{n=1}^{\infty}\left[a_n \int_{-\pi}^{\pi} f(x) \cos nx\, dx + b_n \int_{-\pi}^{\pi} f(x) \sin nx\, dx\right]$$

$$= \frac{a_0}{2}\cdot \pi a_0 + \sum_{n=1}^{\infty}\left(a_n \cdot \pi a_n + b_n \cdot \pi b_n\right)$$

or
$$\frac{1}{\pi}\int_{-\pi}^{\pi} f^2(x)\, dx = \frac{1}{2}a_0^2 + \sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right)$$

Question III. put $x = \pi/2$,

$$f\left(\frac{\pi}{2}\right) = \frac{1}{\pi} + \frac{1}{2}\sin\frac{\pi}{2} + \frac{1}{\pi}\sum_{n=2}^{\infty} \frac{1+(-1)^n}{1-n^2}\cos\left(\frac{n\pi}{2}\right)$$

$$\Rightarrow \sin\frac{\pi}{2} = \frac{1}{\pi} + \frac{1}{2} + \frac{1}{\pi}\left[\frac{2}{-3}(-1) + 0 + \frac{2}{1-16}(1) + 0 + \frac{2}{1-36}(-1) + 0 + \frac{2}{1-64}(1) + \cdots\right]$$

$$\Rightarrow \frac{1}{2} = \frac{1}{\pi} + \frac{2}{\pi}\left[\frac{1}{1\cdot 3} - \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} - \frac{1}{7\cdot 9} + \cdots\right]$$

Multiply by $\pi/2$,

$$\frac{\pi}{4} = \frac{1}{2} + \frac{1}{1\cdot 3} - \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} - \frac{1}{7\cdot 9} + \cdots$$

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Good Luck.